1 (a). $\vec{u}\cdot\vec{v} = 1\times 2 + (-1)\times(-3) + 1\times 1 = 2+3+1 = 6.$

$|\vec{u}| = \sqrt{\vec{u}\cdot\vec{u}} = \sqrt{1^2+(-1)^2+1^2} = \sqrt{3}$

$|\vec{v}| = \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{2^2+(-3)^2+1^2} = \sqrt{4+9+1} = \sqrt{14}$

$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{6}{\sqrt{3}\cdot\sqrt{14}} = \frac{\sqrt{42}}{7}$

(b). $P = \{ax+by+cz+d=0\}$.

$p = (2,2,7) \in P$

$\therefore\ 2a+2b+7c+d=0$ ①

the normal vector of P, $\vec{n}$ ~~is~~ $= (a,b,c)$ $\because \vec{u}\in P.\ \vec{v}\in P.$

$\therefore \vec{n}\perp\vec{u},\ \vec{n}\perp\vec{v}$ $\therefore$ ~~$\vec{n}\cdot\vec{u}=0$ $\vec{n}\cdot\vec{v}=0$.~~

~~$\therefore a-b+c=0$ ②~~ let $\vec{n} = \vec{u}\times\vec{v}$

~~$2a-3b+c=0$ ③~~

$$= \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & -3 & 1 \end{vmatrix}$$

$= (-1+3)i - (1-2)j + (-3+2)k$

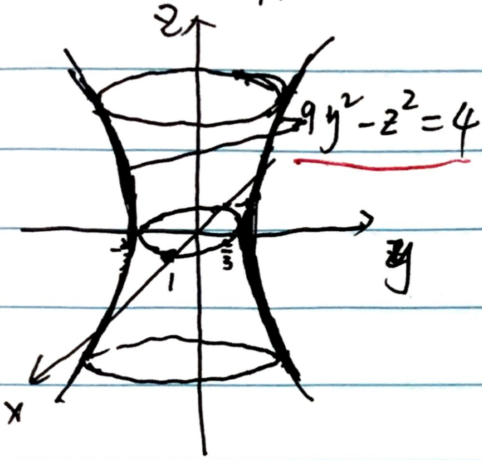
$= 2i + j - k$

$= (2,1,-1)$

$2\times 2 + 2\times 1 + 7\times(-1) + d = 0$ $d = 1$

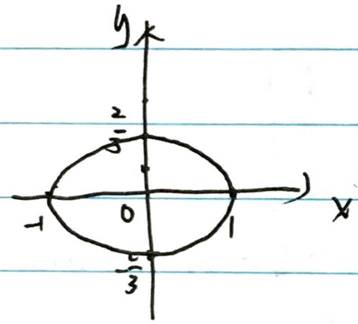
$\therefore P = \{2x+y-z+1=0\}$.

2 (a). Q is a hyperboloid of one sheet.



$$\begin{cases} x=0 \\ 4x^2+9y^2-z^2=4 \end{cases} \Rightarrow 9y^2-z^2=4$$

(b). $$\begin{cases} z=0 \\ 4x^2+9y^2-z^2=4 \end{cases} \Rightarrow 4x^2+9y^2=4 \qquad x^2+\frac{y^2}{\frac{4}{9}}=1$$



(c). $f(x,y) = (X(t), Y(t))$

$x(t) = \cos t \qquad y = \frac{2}{3}\sin t . \qquad x^2+\frac{y^2}{\frac{4}{9}} = \cos^2 t + \frac{\frac{4}{9}\sin^2 t}{\frac{4}{9}} = 1$

$f = (\cos t, \frac{2}{3}\sin t)$

(d). $\cancel{L = \int_0^{2\pi}\sqrt{(\sin t)^2+(\frac{2}{3}\cos t)^2}\,dt}$

$\cancel{= \frac{1}{3}\int_0^{2\pi}\sqrt{5\sin^2 t+4}\,dt} = \frac{1}{3}\int_0^{2\pi}\sqrt{5\cdot\frac{1+\cos 2t}{2}+4}\,dt.$

$\cancel{f = (t, \frac{2}{3}\sqrt{(1-t^2)}) \cup (t, -\frac{2}{3}\sqrt{(1-t^2)}) \quad -1 \le t \le 1}$

$\cancel{= \frac{5}{3}\pi}.$

$\cancel{L = 2\int_{-1}^{1}\sqrt{1+(\frac{2t}{3\sqrt{1-t^2}})^2}\,dt = 2\int_{-1}^{1}\sqrt{1+\frac{4t^2}{9(1-t^2)}}\,dt}.$

$L = \int_0^{2\pi}\sqrt{(-\sin t)^2+(\frac{2}{3}\cos t)^2}\,dt = \frac{1}{3}\int_0^{2\pi}\sqrt{5\sin^2 t+4}\,dt$

$\cancel{= \frac{5}{3}\pi}$

$= \frac{5}{3}\pi$

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3. (a). $K(x) = \frac{1}{|v|}\left|\frac{dT}{dx}\right|$. $\quad p(x) = x\,i + f(x)\,j$. $\quad i=(1,0)\ j=(0,1)$

$$v = p'(x) = 1\cdot i + f'(x)\,j.$$

$$|v| = \sqrt{1+(f'(x))^2}$$

$$T = \frac{v}{|v|} = \frac{1}{\sqrt{1+(f'(x))^2}}\,i + \frac{f'(x)}{\sqrt{1+(f'(x))^2}}\,j$$

$$\frac{dT}{dx} = -\frac{1}{2}\left[1+(f'(x))^2\right]^{-\frac{3}{2}}\cdot 2f'(x)f''(x)\,i + \frac{f''(x)\sqrt{1+(f'(x))^2} - f'(x)\cdot\frac{1}{2}\cdot\frac{2f'(x)\,f''(x)}{\sqrt{1+(f'(x))^2}}}{1+(f'(x))^2}$$

$$= -\frac{f'(x)\,f''(x)}{\left(\sqrt{1+(f'(x))^2}\right)^3}\,i + \frac{f''(x)}{\left(\sqrt{1+(f'(x))^2}\right)^3}\,j$$

$$\left|\frac{dT}{dx}\right| = \sqrt{\frac{\left[-f''(x)\,f'(x)\right]^2}{\left(1+(f'(x))^2\right)^3} + \frac{(f''(x))^2}{\left(1+(f'(x))^2\right)^3}}$$

$$= \frac{|f''(x)|}{1+(f'(x))^2}$$

$$K(x) = \frac{1}{|v|}\left|\frac{dT}{dx}\right| = \frac{|f''(x)|}{\left[1+(f'(x))^2\right]^{\frac{3}{2}}}$$

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(b). radius of the osculating circle at $(1, f(1))$ $\quad r = \frac{1}{K(1)}$

$$f'(x) = e^{2x+4x^3}\cdot(12x^2+2) \qquad f'(1) = 14e^6$$

$$f''(x) = e^{2x+4x^3}(12x^2+2)^2 + e^{2x+4x^3}\cdot 24x$$

$$f''(1) = 14^2e^6 + e^6\cdot 24 = 220e^6.$$

$$r = \frac{(1+196e^{12})^{\frac{3}{2}}}{220e^6}$$

4. (a). $v = p'(t) = \left(\frac{1}{\sqrt{2}}\cos t, -\frac{1}{\sqrt{2}}\sin t, \frac{1}{\sqrt{2}}\right)$.

$|v| = 1$ $\therefore$ t parametrizes the curve C by its arclength

(b). $p(\frac{\pi}{4}) = \left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{2}, \frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{2}, \frac{1}{\sqrt{2}}\times\frac{\pi}{4}\right)$

$= \left(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}\pi}{8}\right)$.

$v(t) = p'(t) = \left(\frac{1}{\sqrt{2}}\cos t, -\frac{1}{\sqrt{2}}\sin t, \frac{1}{\sqrt{2}}\right)$.

$v(\frac{\pi}{4}) = \left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{2}, -\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$= \left(\frac{1}{2}, -\frac{1}{2}, \frac{\sqrt{2}}{2}\right)$.

$L = \left\{\left(\frac{1}{2}+\frac{1}{2}t, \frac{1}{2}-\frac{1}{2}t, \frac{\sqrt{2}\pi}{8}+\frac{\sqrt{2}}{2}t\right) \middle| t \in R\right\}$.

(c). by (b) $v = \left(\frac{1}{\sqrt{2}}\cos t\right)i - \left(\frac{1}{\sqrt{2}}\sin t\right)j + \frac{1}{\sqrt{2}}k$.

$T = \frac{v}{|v|}$

$|v| = \sqrt{\frac{1}{2}\cos^2 t + \frac{1}{2}\sin^2 t + \frac{1}{2}} = 1$.

$\therefore T = v$

$T(\frac{3\pi}{4}) = \left(\frac{1}{\sqrt{2}}\times\left(-\frac{\sqrt{2}}{2}\right), -\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{2}, \frac{1}{\sqrt{2}}\right)$

$= \left(-\frac{1}{2}, -\frac{1}{2}, \frac{\sqrt{2}}{2}\right)$

$N = \frac{dT/dt}{|dT/dt|}$

$\frac{dT}{dt} = \left(-\frac{1}{\sqrt{2}}\sin t\right)i - \left(\frac{1}{\sqrt{2}}\cos t\right)j$

$\left|\frac{dT}{dt}\right| = \sqrt{\frac{1}{2}\sin^2 t + \frac{1}{2}\cos^2 t} = \frac{1}{\sqrt{2}}$.

$\therefore N = (-\sin t)i - (\cos t)j$ $\quad N(\frac{3\pi}{4}) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$

$B = T\times N = \begin{vmatrix} i & j & k \\ -\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{vmatrix}$

$= \left(0-\frac{2}{4}\right)i - \left(0+\frac{2}{4}\right)j + \left(-\frac{\sqrt{2}}{4}-\frac{\sqrt{2}}{4}\right)k$

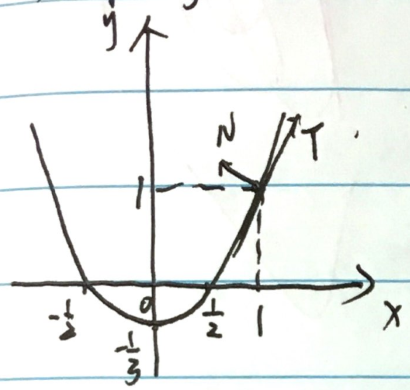
$= \left(-\frac{1}{2}, -\frac{1}{2}, -\frac{\sqrt{2}}{2}\right)$

第 5 題

（答題不得寫在紅綫外）

第

5 (a) $S_1: f(x,y) = 4x^2 - 3y = 1$ $\quad y = \dfrac{4x^2-1}{3}$



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(b). tangent vector $T = (1, \frac{8}{3})$

$N \perp T \qquad N \cdot T = 0$

$\therefore N = (-\frac{8}{3}, 1)$

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6 (a). $\frac{\partial f}{\partial x} = ye^{xy} \cdot \sin(xy+y) + ye^{xy}\cos(xy+y)$

$= ye^{xy}[\sin(xy+y)+\cos(xy+y)]$

$\frac{\partial f}{\partial y} = xe^{xy} \cdot \sin(xy+y) + (x+1)e^{xy} \cdot \cos(xy+y)$

$= e^{xy}[x \cdot \sin(xy+y) + (x+1)\cos(xy+y)]$

(b). $f(0, \frac{\pi}{2}) = e^0 \sin(0+\frac{\pi}{2}) = 1.$

point $p(0, \frac{\pi}{2}, 1)$.

$\left.\frac{\partial f}{\partial x}\right|_{x=0\ y=\frac{\pi}{2}} = \frac{\pi}{2} e^0 [\sin(0+\frac{\pi}{2}) + \cos(0+\frac{\pi}{2})]$

$= \frac{\pi}{2} \times 1 \times (1+0) = \frac{\pi}{2}$

$\left.\frac{\partial f}{\partial y}\right|_{x=0, y=\frac{\pi}{2}} = 0.$

∴ the tangent plane for f at $p(0, \frac{\pi}{2}, 1)$. $P = \{ax+by+cz+d=0\}$

$\frac{\pi}{2}x + \cdots + d = 0$. $\vec{v} = (0,1,0)$, $\vec{u} = (1, 0, \frac{\pi}{2})$, $(a,b,c) = \vec{v} \times \vec{u} = (\frac{\pi}{2}, 0, -1)$

$L_1 = \{(0, \frac{\pi}{2}+t, 1) : t \in R\}$ $L_2 = \{(t, \frac{\pi}{2}, 1+\frac{\pi}{2}t) : t \in R\}$ $L_1 \subset P$, $L_2 \subset P$

∴ $P = \{\frac{\pi}{2}x - z + 1 = 0\}$

$\frac{\pi}{2} \times 0 - 1 + d = 0$, $d = 1$

$x = 3t$, $y = t^2$

$\frac{dx}{dt} = 3$, $\frac{dy}{dt} = 2t$.

(c). $\frac{d(f \circ p)}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$

$= 3t^2 e^{3t^3}(\sin(3t^3+t^2) + \cos(3t^3+t^2)) +$

$2t \cdot e^{3t^3}(3t \sin(3t^3+t^2) + (3t+1)\cos(3t^3+t^2))$

$= e^{3t^3}[9t^2 \sin(3t^3+t^2) + (9t^2+2t)\cos(3t^3+t^2)]$

第 7. 題
（答題不得寫在紅線外）

7. $\frac{dy}{dx} = -\frac{F_x}{F_y}$

$F_x = -2x - y\cos xy$

$F_y = 3y^2 - \cancel{\cos(xy)}\, x\cos xy$

$\therefore \frac{dy}{dx} = \frac{2x + y\cos xy}{3y^2 - x\cos xy}.$